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**RESIDUAL BORE STRESS IN AN
AUTOFRETTAGED CYLINDER CONSTRUCTED
OF A STRAIN HARDENING MATERIAL**

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Ballistic Research Report No. 126

RESIDUAL BORE STRESS IN AN AUTOFRETTAGED CYLINDER
CONSTRUCTED OF A STRAIN HARDENING MATERIAL

Prepared by:
Victor C. D. Dawson

ABSTRACT: The calculation of the residual bore stresses in an autofrettaged cylinder is based upon a theory utilizing a perfectly plastic material. Nadai suggested a technique which would account for strain hardening of the material. The present report compares experimentally measured data for an autofrettaged cylinder, constructed of a strain hardening material, with this theory.

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The report is the result of a continuing effort to provide high strength, high performance guns for launching high velocity aerodynamic models in the ballistics ranges at the Naval Ordnance Laboratory.

J. A. DARE
Captain, USN
Commander

a. e. Seigel
A. E. SEIGEL
By direction

NOLTR 64-25

CONTENTS

	Page
INTRODUCTION.	1
METHOD OF CALCULATION	1
EXPERIMENTAL RESULTS.	4
CONCLUSIONS	6
REFERENCES.	7

ILLUSTRATIONS

Figure

- 1 Average Stress Strain Relation, 4340 Steel
- 2 Measured External Strain Versus Internal Pressure
- 3 τ/γ Versus γ As Constructed from Figure 1
and Equations 10
- 4 Comparison of Calculated and Measured Strain During
Autofrettage of a Work Hardening Material

NOLTR 64-25

List of Symbols

σ	stress (psi)	
r	radius (inches)	
p	internal pressure (psi)	
Y_0	yield strength (psi)	
γ	shear strain	} (microinches/in = μ in/in)
ϵ	strain	
b	outside radius (inches)	
a	inside radius (inches)	
C	plastic-elastic interface radius (inches)	
τ	shear stress (psi)	
E	Young's modulus (psi)	
ω	wall ratio b/a	
N	c/a	
μ	Poisson's ratio	

Subscripts

\dagger	tangential
\wedge	radial

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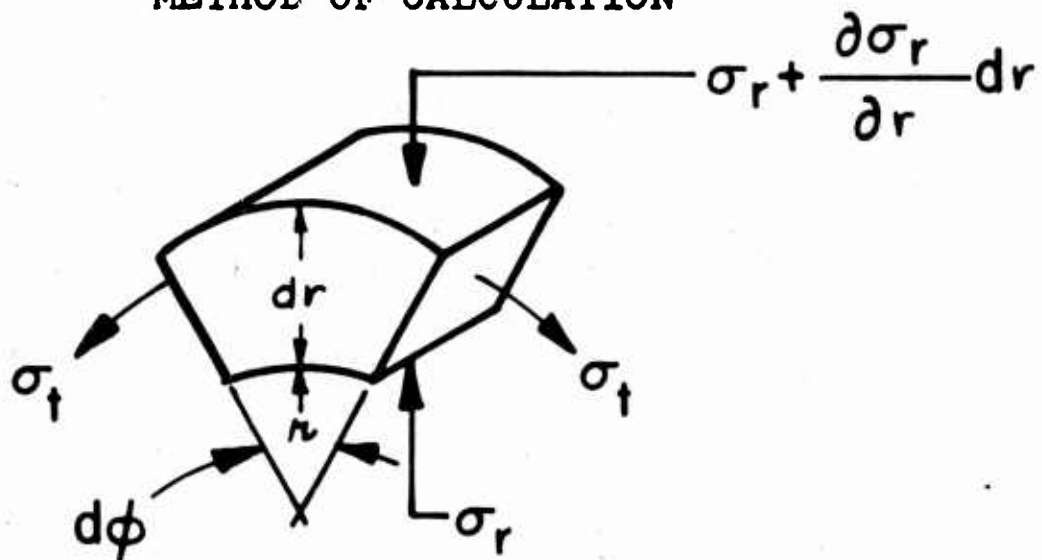
INTRODUCTION

The residual stresses induced by the autofrettage process in high pressure cylinder manufacture have been calculated by numerous investigators, references 1, 2, 3 and 4. Furthermore, the results of these calculations have been experimentally checked by means of the Sachs' technique of residual stress measurement, reference 5, and reasonably good agreement has been obtained for a perfectly plastic material which forms the basis for the theoretical calculations.

When a strain hardenable material is used, the accurate inclusion of work hardening in the analysis greatly complicates the solution. A useful estimate of the pressure expansion curve can be obtained by following the method suggested by Nadai, reference 1.

In the course of an experiment performed to determine the relaxation of residual autofrettage stresses under various thermal treatments, Nadai's method was used to estimate the expansion during autofrettage and the residual stresses induced for a cylinder constructed of a strain hardenable steel. These values were compared directly to the experimental values and the results are presented in this report.

METHOD OF CALCULATION



For the element shown, equilibrium requires that

$$\sigma_t - \sigma_r = r \frac{d\sigma_r}{dr} \quad (1)$$

The boundary conditions for the cylinder with internal pressure only applied are

$$\sigma_r = -p \quad \text{at } r = a$$

$$\sigma_r = 0 \quad \text{at } r = b$$

Thus equation (1) can be written

$$p = \int_a^b (\sigma_t - \sigma_r) \frac{dr}{r} \quad (2)$$

The distortion energy theory for yielding may be written

$$\sigma_t - \sigma_r = \frac{2Y_0}{\sqrt{3}} \quad * \quad (3)$$

If it is assumed that this relation is valid when the material work hardens, then

$$p = \frac{2}{\sqrt{3}} \int_a^b Y \frac{dr}{r} \quad (4)$$

where Y is a function of the equivalent strain and replaces Y_0 in equation (3). Since the state of stress in any element is approximately a hydrostatic tension superposed on a pure shear, it is assumed that Y is a function only of the maximum shear strain. The maximum shear strain for a cylinder is

$\gamma = \epsilon_t - \epsilon_r$ and this is proportional to $\frac{1}{r^2}$ in the elastic region. It is assumed that γ is proportional to $\frac{1}{r^2}$ all through the tube at each stage of the expansion. Hence, writing $\gamma = \frac{b^2 \gamma_b}{r^2}$ where γ_b is the maximum shear strain on the external surface, and $\tau = \frac{Y}{\sqrt{3}}$

*where Y_0 is the yield strength for a perfectly plastic material.

$$p = \int_{\gamma_b}^{\gamma_a} \tau(\gamma) \frac{d\gamma}{\gamma} \quad (5)$$

where $\gamma_a = \frac{b^2 \gamma_b}{a^2}$ is the shear strain at the bore.

The tangential strain in a cylinder during autofrettage is given by

$$\epsilon_t = \frac{u}{r} = \frac{Y_0 N^2}{\sqrt{3} E \omega^2} \left[(1 - 2\mu) + (1 + \mu) \frac{b^2}{r^2} \right] \quad (6)$$

Also since $\epsilon_t = \frac{du}{dr}$

$$\epsilon_r = \frac{Y_0 N^2}{\sqrt{3} E \omega^2} \left[(1 - 2\mu) - (1 + \mu) \frac{b^2}{r^2} \right] \quad (7)$$

Thus

$$\gamma = \epsilon_t - \epsilon_r = \frac{Y_0 N^2}{\sqrt{3} E \omega^2} 2(1 + \mu) \frac{b^2}{r^2} \quad (8)$$

At $r = b$, from (6)

$$\epsilon_{t_b} = \frac{Y_0 N^2}{\sqrt{3} E \omega^2} (2 - \mu)$$

so that by combining this result with (8) at $r = b$

$$\gamma_b = \frac{2(1 + \mu)}{(2 - \mu)} \epsilon_{t_b} \quad (9)$$

By means of equations (9) and (5) it is now possible to calculate the pressure-expansion curve by numerical means provided that a shear stress-shear strain curve is available for the material.

EXPERIMENTAL RESULTS

A 1 inch OD x 0.5 inch ID closed end test cylinder was constructed of AISI 4340 and heat treated for a nominal yield of 120,000 psi. This cylinder, which was 43 inches long, was pressurized to 95,000 psi and both the longitudinal and transverse strains on the outside surface of the cylinder were measured during autofrettage (six transverse gages and three longitudinal gages were used). The autofrettaged cylinder was then sectioned into 3½-inch long specimens. Two of these were used to determine the residual stress distribution that was introduced by the autofrettage process. The method used for this was the Sachs' boring out technique, reference 5.

Figure 1 shows the stress-strain curve obtained in tension for the AISI 4340 steel used in the experiment. This curve represents an average of three separate tensile tests (two specimens were cut in the transverse direction and one in the longitudinal direction). The maximum difference in stress, for a given strain, between the three specimens was less than 5 percent.

Figure 2 shows the pressure vs. tangential strain at the outside surface of the 43-inch long cylinder during autofrettage. Based upon the .2 percent offset yield of 120,000 psi, as given in figure 1, the pressure at which yielding should begin in the cylinder is about 52,000 psi. In figure 2 this is indicated by a departure of the pressure-strain curve from a straight line at approximately 52,000 psi.

In order to calculate the pressure-strain curve by means of equations (9) and (5) it is necessary to have shear stress-strain data. Although this was not obtained experimentally, the relationships (as given in reference 1) between the shear stress and shear strain and the tensile stress and tensile strain are

$$\tau = \frac{\sigma_{\text{tension}}}{\sqrt{3}}$$

and

$$\gamma = \frac{3\epsilon_{\text{tension}}}{2} \quad (10)$$

With these relations and figure 1 it is now possible to calculate τ / γ as a function of γ as shown in figure 3. By assuming a value for ϵ_{t_b} and calculating γ_b from (9) and $\gamma_a = \frac{b^2}{a^2} \gamma_b$ it is possible to numerically integrate the curve shown in figure 3 between these limits of γ_a and γ_b and obtain the pressure p as given by equation (5). Thus for each assumed value of ϵ_{t_b} , a value of p necessary to provide the assumed ϵ_{t_b} value is obtained. The values of pressure obtained in this manner for six assumed values of ϵ_{t_b} are given in Table 1.

Figure 4 is a plot of the numerically calculated values of Table 1 in comparison to the experimentally measured values. The agreement between the calculated and experimental data is with ± 3.5 percent. Inasmuch as

$$\sigma_t - \sigma_r = \frac{2\gamma}{\sqrt{3}} = 2\tau$$

and

$$\sigma_{ra} = - \int_{\gamma_b}^{\gamma_a} \tau \frac{d\gamma}{\gamma}$$

$$\sigma_{ta} = 2\tau_a - \int_{\gamma_b}^{\gamma_a} \tau \frac{d\gamma}{\gamma}$$

Evaluating τ at $\gamma = \gamma_a$ from the shear stress curve approximated from the tensile test data and letting

$$\int_{\gamma_b}^{\gamma_a} \tau \frac{d\gamma}{\gamma} = 93100 \text{ p.s.i.}$$

gives a value of $\sigma_t = 53,500 \text{ psi}$. This represents the tangential stress at the bore during the autofrettage process. The elastic equivalent of the bore tangential stress at a

NOLTR 64-25

pressure of 93,100 psi is 155,000 psi. If this value is subtracted from the 53,500 psi, then the residual tangential stress at the bore predicted by taking account of work hardening is -101,000 psi. The average value obtained by the Sach's technique of measurement was -97,500 psi, i.e., within 4 percent of the calculated value.

CONCLUSIONS

Nadai's method of calculating the expansion of a cylinder, constructed of a work hardenable material, has been experimentally verified. The results of the analytical calculation and experimental test agreed within 3.5 percent. Furthermore, the residual tangential bore stress can be calculated. The value obtained agreed within 4 percent with the experimentally measured value.

NOLTR 64-25

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NOLTR 64-25

Table 1

ϵ_t_b μ in/in	γ_b μ in/in	γ_a μ in/in	Pressure (psi)
900	1,380	5,520	47,200
1,200	1,835	7,340	59,600
1,400	2,140	8,560	66,500
2,000	3,060	12,240	86,500
2,500	3,820	15,300	89,400
2,665	4,060	16,280	93,100

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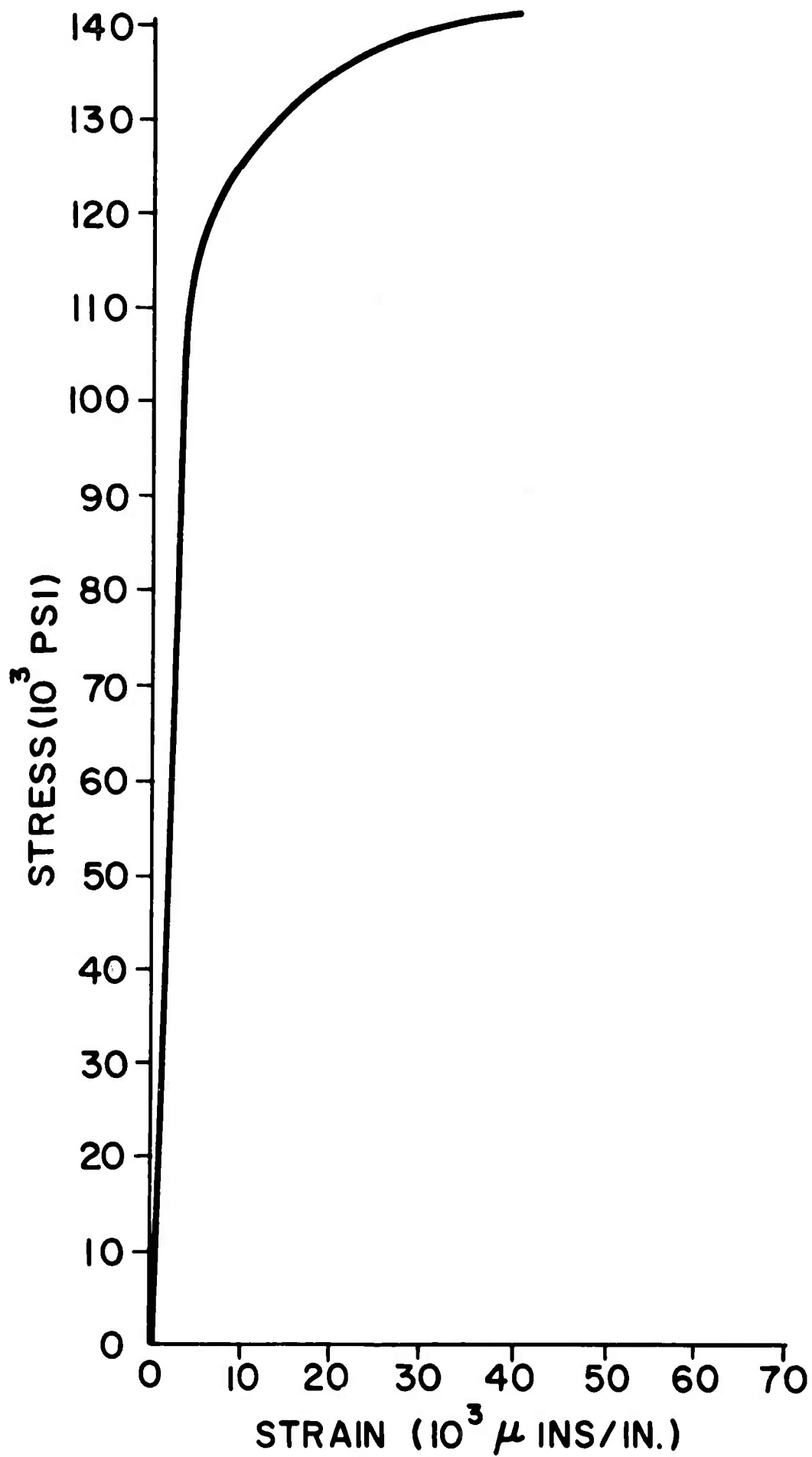


FIG. 1 AVERAGE STRESS STRAIN
RELATION, 4340 STEEL

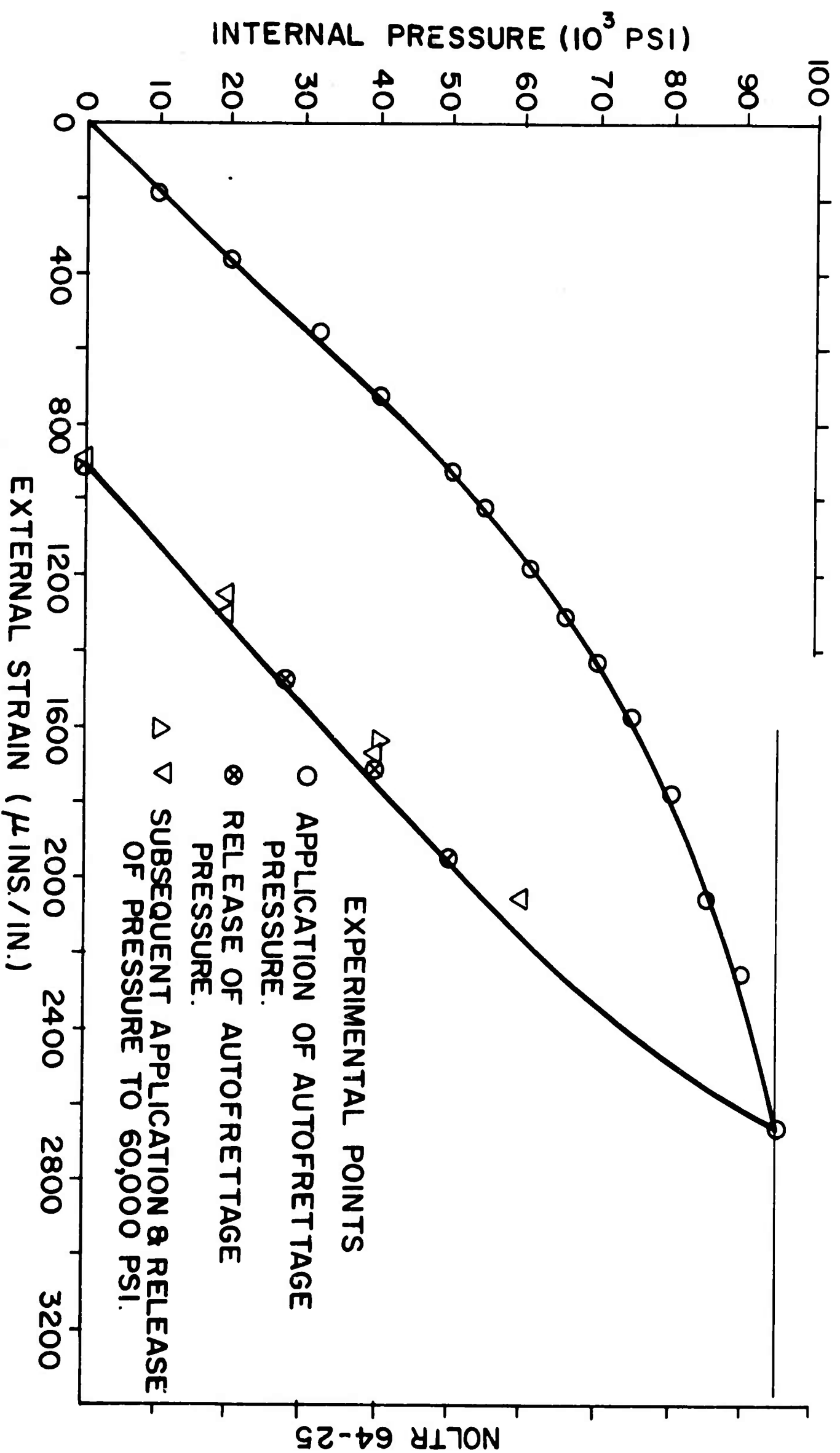


FIG. 2 MEASURED EXTERNAL STRAIN VERSUS
INTERNAL PRESSURE.

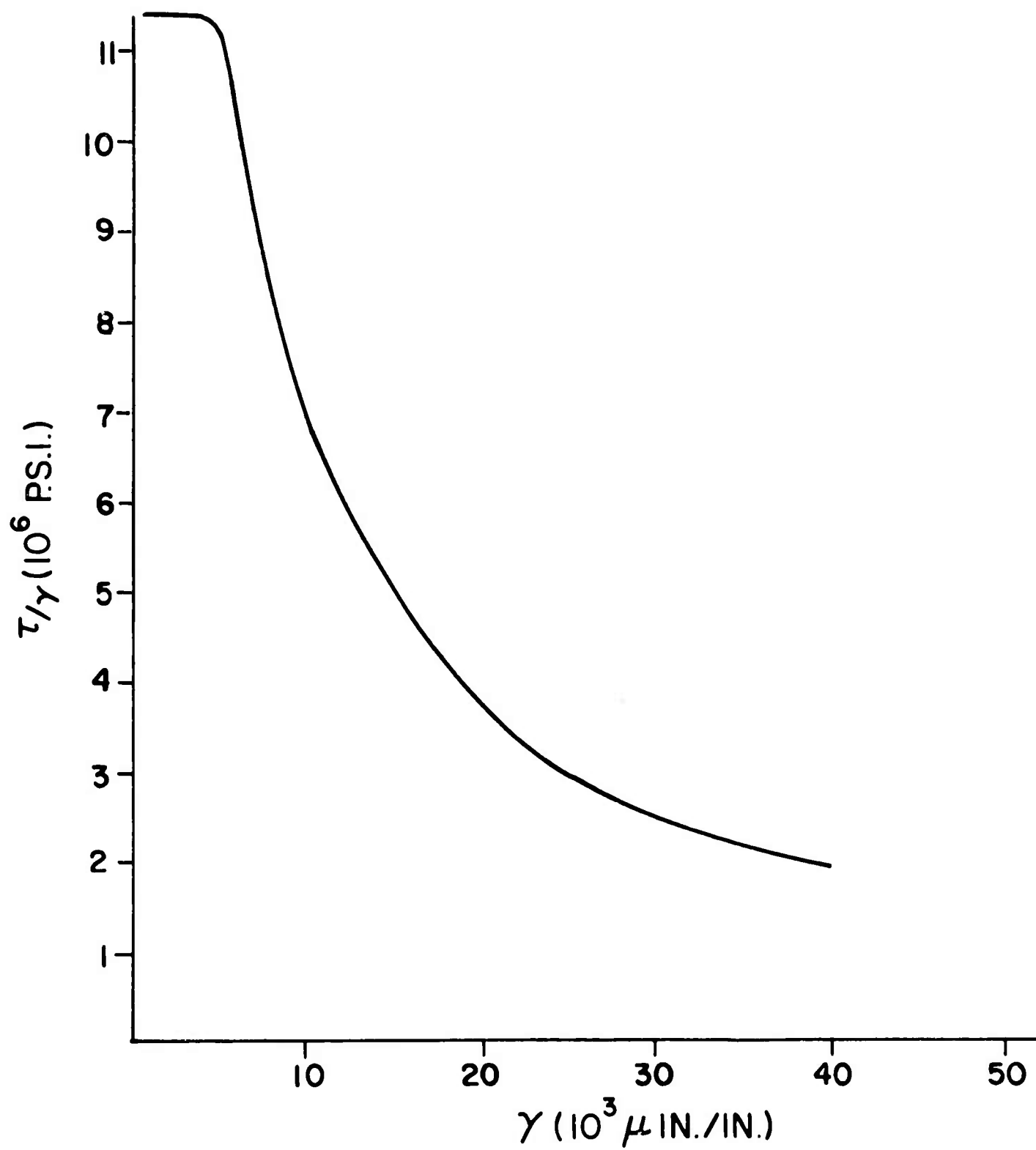


FIG. 3 τ/γ VERSUS γ AS CON-
STRUCTED FROM FIG. 1
AND EQTS. 10

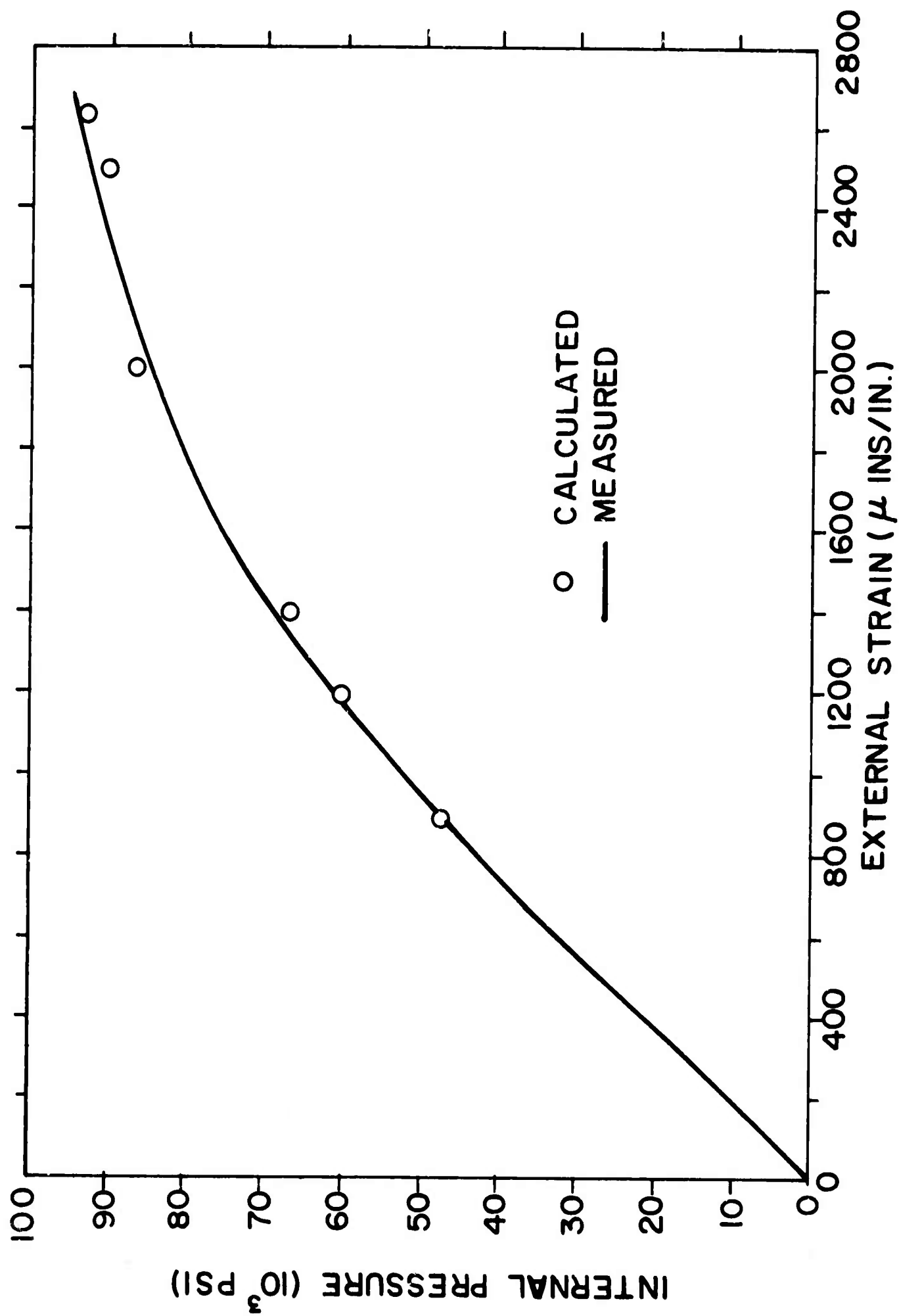


FIG. 4 COMPARISON OF CALCULATED & MEASURED STRAIN DURING AUTOFRETTAGE OF A WORK HARDENING MATERIAL.

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